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COMMENT

A comment on the new Dirac-like spin-$\frac{3}{2}$ wave equation of Jayaraman

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Abstract. The new spin-$\frac{3}{2}$ wave equation of Jayaraman is shown to be an eight-dimensional form of the Dirac equation.

There has been a recent article in this journal (Jayaraman 1976, to be referred to as I) concerning a derivation of a 'new linear Dirac-like spin-$\frac{3}{2}$ wave equation'. The new equation of I was derived from a Schrödinger-type wave equation (Weaver *et al* 1964, Mathews 1966a, b, 1967a, b, Seetharaman *et al* 1971, Jayaraman 1973a, b, 1975) in which the wavefunction has eight components and transforms according to the $(0, \frac{3}{2})\oplus(\frac{3}{2}, 0)$ representation of the homogeneous Lorentz group.

The purpose of this comment is to point out that the procedure employed in I seems to produce the Dirac equation itself in an eight-dimensional form rather than a spin-$\frac{3}{2}$ wave equation.

To see this point in a transparent way, it is simplest to change the representation of the matrices used in I which were

$$t_1 = 2l_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{bmatrix}, \qquad t_2 = 2l_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 1 & 0 & 0 & 0 \\ 0 & -\mathrm{i} & 0 & 0 \end{bmatrix},$$

$$t_3 = 2l_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -\mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

with the unitary matrix

$$U = \frac{1}{2}\begin{bmatrix} 1 & 1 & -\mathrm{i} & -1 \\ 1 & 1 & \mathrm{i} & 1 \\ 1 & -1 & \mathrm{i} & -1 \\ -1 & 1 & \mathrm{i} & -1 \end{bmatrix} \tag{2}$$

one finds the new representation of the t_i ($i = 1, 2, 3$) to be

$$Ut_iU^\dagger = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix} \equiv \Sigma_i \tag{3}$$

where the σ_i are the usual representation of the Pauli matrices with

$$\sigma_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \qquad \sigma_2=\begin{bmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{bmatrix}, \qquad \sigma_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}. \tag{4}$$

In this representation, the Dirac-like wave equation found in I has the form

$$\left(\begin{bmatrix}0 & \boldsymbol{\Sigma}\cdot\boldsymbol{P}\\ \boldsymbol{\Sigma}\cdot\boldsymbol{P} & 0\end{bmatrix}+\begin{bmatrix}I & 0\\ 0 & -I\end{bmatrix}m\right)\begin{pmatrix}\chi_\mathrm{u}\\ \chi_\mathrm{l}\end{pmatrix}=\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix}\chi_\mathrm{u}\\ \chi_\mathrm{l}\end{pmatrix} \tag{5}$$

or

$$\boldsymbol{\Sigma}\cdot\boldsymbol{P}\chi_\mathrm{l}+m\chi_\mathrm{u}=\mathrm{i}\frac{\partial}{\partial t}\chi_\mathrm{u} \qquad \boldsymbol{\Sigma}\cdot\boldsymbol{P}\chi_\mathrm{u}-m\chi_\mathrm{l}=\mathrm{i}\frac{\partial}{\partial t}\chi_\mathrm{l} \tag{6}$$

for the upper and lower four components χ_u and χ_l. It is clear, for example, that in a constant, external magnetic field, the eigenvalues of equation (5) are the same (but twice as many) as those of a spin-$\frac{1}{2}$ Dirac particle. In particular, the non-relativistic limit produces

$$H_\mathrm{nr}=\frac{\pi^2}{2m}-2\frac{q}{2m}(\boldsymbol{\Sigma}/2)\cdot\boldsymbol{B} \tag{7}$$

with $\boldsymbol{\pi}=\boldsymbol{P}-q\boldsymbol{A}$, $\boldsymbol{A}=\frac{1}{2}\boldsymbol{B}\times\boldsymbol{X}$ and q the charge. Since the non-relativistic magnetic moment interaction is $-gq\boldsymbol{S}\cdot\boldsymbol{B}/2m$ for spin-s with spin matrices $\boldsymbol{S}$ the identification $\boldsymbol{S}=\boldsymbol{\Sigma}/2$ demonstrates the spin-$\frac{1}{2}$ content of equations (4) and (6). The only way to avoid this conclusion is to reject the minimal substitution $\boldsymbol{P}\to\boldsymbol{\pi}$, and to use some complicated interaction in its place. This kind of manipulation would, however, negate any advantage of a Dirac-like wave equation, for which the price of complicated wavefunction transformation has already been paid.

References

Jayaraman J 1973a *Nuovo Cim.* A **13** 877–96
—— 1973b *Nuovo Cim.* A **14** 343–62
—— 1975 *J. Phys. A: Math. Gen.* **8** L1–4
—— 1976 *J. Phys. A: Math. Gen.* **9** L131–6
Mathews P M 1966a *Phys. Rev.* **143** 978–85
—— 1966b *Phys. Rev.* **143** 985–9
—— 1967a *Phys. Rev.* **155** 1415–20
—— 1967b *J. Math. Phys. Sci.* **1** 197–206
Seetharaman M, Jayaraman J and Mathews P M 1971 *J. Math. Phys.* **12** 835–40
Weaver D L, Hammer C L and Good R H Jr 1964 *Phys. Rev.* **135** B241–8